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APPLICATION OF LAGUERRE-VORONOI DIAGRAMS FOR CONSTRUCTION AND OPTIMIZATION OF THE EMERGENCY NOTIFICATION SYSTEM INFRASTRUCTURE FOR THE POPULATION

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Key words: *Laguerre-Voronoi diagram, emergency alert systems, source placement optimization, acoustic waves, sound pressure.*

The relevance of the article is due to the need to develop and improve methods for optimal placement of sound signal sources of emergency alert systems to ensure the effectiveness of their functioning. The development of new approaches will reduce the costs of maintaining and placing such systems, improve their deployment time and provide the population of the alerting area with a normative and more comfortable level of acoustic pressure of sound signals.

The purpose of the study is to develop an algorithm for using Laguerre-Voronoi diagrams to place sound signal sources of the emergency alert system, taking into account acoustic and other parameters that affect the quality of its operation.

The article presents the results of modeling and an algorithm for using Laguerre-Voronoi diagrams to optimize the placement of sound signal sources in emergency alert systems. The proposed methodology allows you to model and determine coverage areas taking into account the effective range of alert sources, building density, terrain features and acoustic characteristics of the area. The developed algorithm takes into account the peculiarities of sound wave propagation in an environment with different conditions of acoustic wave absorption, taking into account the peculiarities of the territory's development and the height of the sources, which affect their effective range.

The possibilities of applying the proposed approach are demonstrated in the example of calculating the locations of sound signal sources of the warning system for a model city area. The possibilities of taking into account the acoustic characteristics of buildings in the warning area and ensuring the parameters of the regulatory sound pressure level are shown. An algorithm for optimizing the use of the ranges of sound signal sources is presented.

ЗАСТОСУВАННЯ ДІАГРАМ ЛАГЕРРА – ВОРОНОГО ДЛЯ ПОБУДОВИ ТА ОПТИМІЗАЦІЇ ІНФРАСТРУКТУРИ СИСТЕМИ ЕКСТРЕНОГО ОПОВІЩЕННЯ НАСЕЛЕННЯ

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Ключові слова: *діаграма Лагерра – Вороного, системи екстреного оповіщення, оптимізація розміщення джерел, акустичні хвилі, звуковий тиск.*

Актуальність статті обумовлена потребою розробки й удосконалення методів оптимального розміщення джерел звукових сигналів систем екстреного оповіщення населення для забезпечення ефективності їх функціонування. Розробка нових підходів дасть змогу зменшити витрати на обслуговування та розміщення таких систем, покращити час їх розгортання та забезпечити для населення території оповіщення нормативний і більш комфортний рівень акустичного тиску звукових сигналів.

Метою дослідження є розробка алгоритму застосування діаграм Лагерра – Вороного для розміщення джерел звукових сигналів системи екстреного оповіщення населення з урахуванням акустичних та інших параметрів, що впливають на якість її роботи.

У статті наведено результати моделювання й алгоритм використання діаграм Лагерра – Вороного для оптимізації розміщення джерел звукових сигналів у системах екстреного оповіщення населення. Запропонована методика дає змогу моделювати та визначати зони покриття з урахуванням ефективної дальності дії джерел оповіщення, щільності забудови, особливостей рельєфу й акустичних характеристик місцевості. Розроблений алгоритм враховує особливості поширення звукових хвиль у середовищі з різними умовами поглинання акустичних хвиль з огляду на особливості забудови території та висоту розміщення джерел, які впливають на їх ефективну дальність.

Можливості застосування запропонованого підходу продемонстровано на прикладі розрахунку місць розміщення джерел звукових сигналів системи оповіщення для модельної території міста. Показано можливості урахування акустичних характеристик будівель території оповіщення та забезпечення параметрів нормативного рівня звукового тиску. Наведено алгоритм оптимізації використання радіусів дії джерел звукових сигналів.

Introduction. Today, the efficiency of emergency alert systems for the population largely depends on the effective placement of sound signal sources. High-quality coverage of a territory with alert signals requires consideration of its topographical features, building density, terrain influence, acoustic characteristics, and other parameters, making this a complex task. Insufficient coverage leads to the risk of alert gaps, while excessive overlap may result in acoustic pressure exceeding regulatory levels. The propagation characteristics of sound waves are also crucial for effective system modeling. The intensity of a sound signal gradually decreases as the distance from the alert source increases due to factors such as environmental absorption and the density of the area's development. This attenuation, caused by the unique features of the terrain, can significantly reduce the effective coverage radius. Additionally, the height at which the sources are placed affects their efficiency by mitigating the impact of terrain obstacles and buildings. An essential condition for the safe operation of alert systems is compliance with regulatory sound pressure levels within the frequency range of 63 Hz to 8000 Hz.

Therefore, the issue of further improving and developing methods for optimizing the placement of emergency alert sources remains highly relevant. This includes accounting for critical factors such as the sources’ effective range, signal attenuation due to absorption, installation height, and compliance with acoustic parameters to regulatory standards.

Problem Statement. The aim of the study is to develop an algorithm for applying Laguerre–Voronoi diagrams to address the problem of rational placement of sound signal sources in emergency alert systems for the population, taking into account acoustic characteristics and additional parameters that may influence the system's performance quality. The use of Laguerre–Voronoi diagrams will enable the calculation of effective influence zones for each sound signal source, ensure uniform coverage of the alert area, and minimize acoustic discomfort in compliance with established norms. Furthermore, it will facilitate the optimization of the placement of sound signal sources in emergency alert systems.

Literature Review. The problem of optimizing the placement of sound sources with different signals to ensure effective coverage of an alert area belongs to the class of problems within the mathematical theory of optimal set partitioning (OSP) in n-dimensional Euclidean space.

The origins, dynamics, and prospects for further development and application of the OSP theory in n-dimensional Euclidean space are outlined in [1]. Examples of applying OSP theory to various theoretical optimization problems and solutions to the generalized location-allocation problem are provided.

A generalization of solutions for continuous OSP problems based on partition quality criteria, ensuring a specific type of Voronoi diagram, is presented in [2]. Mathematical and algorithmic tools for constructing both existing and new Voronoi diagrams are proposed.

By synthesizing OSP solution methods with neuro-fuzzy technologies and modifying Shor’s r-algorithm for non-smooth optimization problems, a method for solving OSP problems with fuzzy parameters in constraints is proposed in [3]. The effectiveness of the approach is demonstrated through a model problem that partitions a set into three subsets with fuzzy constraint parameters.

The potential applications of OSP theory for continuous problems in n-dimensional Euclidean space to artificial intelligence tasks are explored in [4]. Pattern recognition problems under certainty and uncertainty are formulated, with special attention to fuzzy Voronoi diagrams. Examples of constructing fuzzy Voronoi diagrams with optimal generator point placement are included.

The formulation and numerical algorithm for solving a continuous linear OSP problem for multiplex partitioning of bounded two-dimensional sets – modeling optimal placement of service centers with overlapping service zones – are detailed in [5]. Specific cases are demonstrated: a fixed set of centers, optimization of their coordinates within a given set, and adding new centers with redistributing service zones in an updated network.

A methodology for applying OSP theory in n-dimensional Euclidean space to solve location-allocation problems, described through infinite-dimensional OSP models, is presented in [6].

The results of applying OSP models to optimize the placement of bank service units providing various services while partitioning client regions into service zones for customer flow optimization are detailed in [7]. This problem is characterized as a continuous nonlinear multi-product OSP problem with constraints on subset center coordinates. The analytical solution includes optimization parameters.

Study [8] proposes the use of generalized Voronoi diagrams based on Laguerre geometry for managing heterogeneous sensor networks. This approach adapts the OSP model for sensors with varying radii and directions of influence.

An algorithm for the optimal placement of sensor nodes in territories with various obstacles, based on modified Voronoi diagrams, is presented in [9]. This method accounts for noise and terrain variations.

The methodology for reducing the construction of optimal solutions for infinite-dimensional location-allocation problems to solving operator equations with parameters derived from an auxiliary finite-dimensional non-differentiable optimization problem is described in [10].

A review of cooperative control strategies for sensor networks to achieve complete territorial coverage, utilizing modified Voronoi diagram metrics, along with experimental results, is presented in [11].

The results of studying the acoustic properties of absorbing materials and the specifics of sound wave absorption in environments with various acoustic characteristics are discussed in [12].

Key algorithms for computing power diagrams of sets, features of working with Laguerre–Voronoi diagrams, and the challenges and opportunities of their practical use are outlined in [13].

Research Methods and Algorithms. To address the problem of rational placement of sound signal sources for the emergency alert system, it is proposed to use the Laguerre–Voronoi diagram [8; 13]. The Laguerre–Voronoi diagram, also referred to as the ownership diagram, represents a generalization of the standard Voronoi diagram for partitioning a subset of points into a system of circles, enabling its use for complete coverage of a given area with circles of varying radii. Thus, in this case, the Laguerre–Voronoi diagram will represent a system of circles, each reflecting the area of sound signal propagation from an alert source located at the center of the circle, ensuring complete coverage of the alert territory while taking into account its specific features and the possibility of using sound sources of different power levels.

In this case, the construction of the ownership diagram depends on the coordinates of the locations of the sound signal sources and the corresponding area of the alert territory and is determined by the generalized ownership (distance) function [13]:

$$w(P, C_i) = r^2 - d^2, \qquad (1)$$

where $w(P, C_i)$ – is the membership function of point P to the circle with center C_i;
r – is the radius of circle C_i; d – is the distance from point P to the center of circle C_i, $d^2 = (x - x_i)^2 + (y - y_i)^2$.

The values of function (1) allow determining the distribution zones of the territory among the alert sources according to the following criteria

$$P(x, y) C_i(x_i, y_i) \ \text{при} \ w(P, C_i) \geq 0, \qquad (2)$$

$$P(x, y) C_i(x_i, y_i) \ \text{при} \ w(P, C_i) < 0, \qquad (3)$$

Thus, applying the membership function (1) with criteria (2) and (3) allows partitioning the entire set of points in the territory into subsets of influence zones of the nearest alert sound sources with specified power, forming a system of convex polygons. Each polygon represents the coverage area of a specific source. The coverage area of an alert source is a subset of all points P, for which the membership function relative to circle C_i is greater than that for any other circle:

$$Cell(C_i) = \cap_{j \neq i} \{P | w(P, C_i) \geq w(P, C_j)\}, \qquad (4)$$

where $Cell(C_i)$ – is the subset of points in the influence zone of source C_i;
$\cap$ – intersection of sets of points that satisfy the condition of minimal distance.

The boundary of the influence zones of two alert sources is determined using the radical axis, which is the geometric locus of all points for which the membership function relative to the two circles has equal values:

$$\|P - C_i\|^2 - r_i^2 = \|P - C_j\|^2 - r_j^2 \qquad (5)$$

where $\|P - C_i\|^2$ – is the squared distance between point P and center C_i;
$\|P - C_j\|^2$ – is the squared distance between point P and center C_j;
r_i^2, r_j^2 – are the squares of the radii of the respective circles.

According to the proposed algorithm for constructing the rational infrastructure of the emergency alert system using ownership diagrams, it is necessary to:

1) Define the parametric coefficients of acoustic wave propagation efficiency from the alert sound source locations, taking into account additional parameters such as the height of the sources and the density of urban development;

2) Calculate the coverage zones for the sound sources considering the parameters of the selected sound generation equipment models, their power, and the specific features of the coverage area;

3) Determine the required number of sound sources to fully cover the alert territory and the coordinates of their placement (x_i, y_i);

4) Partition the set of points in the alert territory into coverage zone subsets using formula (1) and construct radical axes of the ownership diagram for each pair of sources using formula (5).

The specific features of the alert territory, elevation changes, and other important parameters are considered through modifications of the radii r_i and corresponding parametric coefficients.

The Laguerre–Voronoi diagram constructed in this way determines the coverage zones for each alert sound source, which are bounded by radical axes. The influence zones of each source and the radical axes are projected onto the plane in the form of corresponding circles and straight lines, as shown in Fig. 1.

Additionally, calculating specific coverage zones for alert sources requires consideration of their acoustic characteristics, such as sound pressure, distance, the speed of acoustic wave propagation, signal absorption by the environment, and its impact on signal range, building density, and the height of source placement. For example, the intensity of a sound signal decreases proportionally to the square of the distance from the generation source and is calculated as follows:

$$p(x, y) = \frac{p_0}{4\left[(x - x_i)^2 - (y - y_i)^2\right]}, \qquad (6)$$

where $p(x, y)$ – is the intensity of acoustic pressure at point (x, y) in the influence zone of source $C_i(x_i, y_i)$;
p_0 – is the intensity of acoustic pressure at the source of generation.

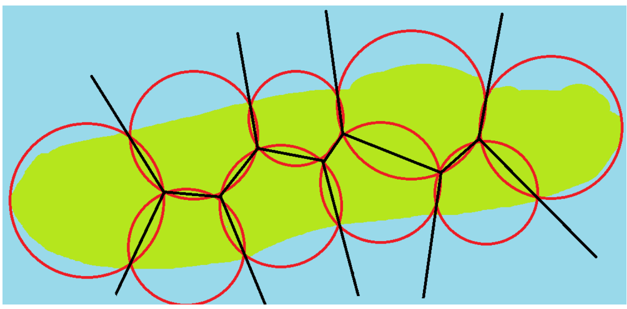

Fig. 1. Example of an ownership diagram with the placement of 9 sources

In some cases, the radius of influence of alert sources allows accounting for sound signal attenuation. However, for more accurate consideration of real conditions, it is advisable to include the effect of acoustic absorption by objects in the alert territory using the following formula (5):

$$p_{eff}(d) = p(d)e^{-d}, \quad (7)$$

where $p_{eff}(d)$ – is the effective sound pressure at distance d;
α – is the absorption coefficient of the environment.

By using the exponential attenuation formula (7), it is possible to pre-calculate the influence of atmospheric density, and the features and materials of buildings in the alert territory.

Another important parameter influencing the radius of alert source operation is the height of their placement. The higher the alert source is positioned, the better and larger its effective range, as the influence of terrain obstacles and buildings decreases. This effective radius can be calculated as follows:

$$r_{eff} = r_0 + h * k, \quad (8)$$

where r_0 – is the base radius of the alert source, *m*;
h – is the height of source placement, *m*;
k – is the coefficient of height influence on the radius.

Another critical parameter is the permissible level of acoustic pressure, ensuring a balance between alert efficiency and the safety of sound signal levels for the population. Relevant normative parameters for permissible sound pressure levels for humans at various frequencies are presented in Table 1.

Using these values ensures the necessary sensitivity of the population to the functioning of the alert system while minimizing its impact on their health.

After considering all parameters and constraints, it is possible to optimize the placement of the alert points. This requires calculating and evaluating the proper signal level in the source's influence zone:

$$p_{eff}(x, y) \geq p_{min}, \quad (9)$$

where p_{min} – is the minimum sound pressure sufficient for human perception;
where p_{min} – is the minimum sound pressure sufficient for human perception;
p_{min} is considered sufficient at the level $p_{min} \geq 50$ dB;
$p_{eff}(x, y)$ –is the effective sound pressure at the point (x, y) in the alert zone;
dA – is the elementary area.

The verification of ensuring the proper signal level within the coverage area of each source is carried out using formula (9). If the result for the points of placement of the radical axes is significantly greater than p_{min}, the system can be optimized by reducing the power of the warning source, its placement height, or another parameter that would allow reducing the resulting influence of two sources and optimizing energy costs.

Analysis of the results. The application of the proposed approach will be illustrated by the example of determining the locations of sound signal sources for a warning system in a small town or a district of a city, which is approximated by a rectangle of 30 by 20 kilometers. The coordinates of its points will be: (0, 0), (30000, 0), (0, 20000), (30000, 20000). This rectangle has three types of buildings, each with its own environmental absorption coefficient: central zone $\alpha = 0.15$, peripheral zone $\alpha = 0.1$, and industrial zone $\alpha = 0.05$.

For the warning sources, the initial sound pressure level is $p_0 = 100$ dB. Accordingly, the minimum and maximum values are: $p_{min} = 50$ dB; $p_{max} = 110$ dB.

The number of sources to be placed – 6.

The radius of action of the sources without considering height is: $r_0 = 6800$ m.

The height influence coefficient on the radius of the source's action is: $k = 0.5$.

The rectangle is divided into 6 zones, approximated to squares, with one source placed at the center of each zone. After placing the sources, they will have a certain height depending on the area or location. In the zero approximation, it is assumed that each source has a placement height of 50 m.

The city's area is roughly divided into three density zones, and therefore each source will have a different absorption coefficient α. The placement coordinates of the sources in the zero approximation are as follows:

Source 1: (5000 , 5000), peripheral zone: $\alpha = 0.05$.
Source 2: (15000, 5000), industrial zone: $\alpha = 0.1$.
Source 3: (25000, 5000), central zone: $\alpha = 0.15$.
Source 5: (5000 , 15000), peripheral zone: $\alpha = 0.05$.
Source 6: (15000, 15000), industrial zone: $\alpha = 0.1$.
Source 7: (25000, 15000), central zone: $\alpha = 0.15$.

For each placement variant, the coverage radius is determined considering the building density and the source height using formula (8):

$r_{eff1} = 6800 * (1 - 0.05) * (1 + 0.5 * 0.5) = 8060$ m,
$r_{eff2} = 6800 * (1 - 0.5) * (1 + 0.5 * 0.5) = 7650$ m,
$r_{eff3} = 6800 * (1 - 0.15) * (1 + 0.5 * 0.5) = 7220$ m,
$r_{eff4} = 6800 * (1 - 0.05) * (1 + 0.5 * 0.5) = 8060$ m,
$r_{eff5} = 6800 * (1 - 0.5) * (1 + 0.5 * 0.5) = 7650$ m,
$r_{eff6} = 6800 * (1 - 0.15) * (1 + 0.5 * 0.5) = 7220$ m,

Now, for each source, the influence zone is calculated. For each point of the warning territory, the membership function value (1) is calculated, and based on the results, it is assigned to the coverage

Table 1

Normative parameters of sound pressure for various frequency values

Vibration parameters	Numerical values of sound vibration parameters							
Average frequencies of octave bands, Hz	63	125	250	500	1000	2000	4000	8000
Limit sound pressure level, dB	110	94	87	81	78	75	73	71

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zone of the source for which $w(P, C_i)$ is minimal. This step might require many calculations, so the territory area is divided into a grid with a step of 1000 m. The calculations are performed only for the central point, thus the entire cell is included in the influence zone. The algorithm's accuracy depends on the grid step size. The smaller the step, the higher the accuracy.

To determine the boundaries of the coverage zones of each source, the coordinates of the radical axes are calculated using formula (2). These will be straight lines passing through points that have the same power relative to the two notification sources. As a result, zones of influence are formed, limited by the radical axes. For example, for sources 1 and 2, the radical axis will be vertical, $x = 10000$, since $y_1 = y_2$.

For the formed coverage zones, a check is performed to ensure complete partitioning of the warning territory between the sources. The distance between two sources must be less than the sum of their radii. For example, for sources 1 and 2, $d_{12} = 10000$ m, which is less than $r_{eff1} + r_{eff2} = 8060 + 7650 = 15710$ m.

Next, optimization of the radii is performed:

1) If the distance between two sources is less than the sum of their action radii, optimization is considered by reducing the action radius of the warning sources, considering constraints.

2) If the distance between two sources is greater than the sum of their radii, optimization is carried out by increasing the placement height of the warning source or its power within the norm.

3) If the distance is too large, the possibility of placing an additional warning source and forming new coverage zones is considered.

In this case, most sources had excessive overlap, so optimization was performed by reducing the placement height of all sources, and new radii were calculated, as shown in Table 2.

A check was also carried out for the obtained variant of placing sound signal sources to ensure the required sound pressure level according to criterion (9).

For example, for Source 1, the sound pressure values at points in the coverage zone:

Point (0, 0), distance from source to point $d = 7071$:

$p_{eff}(700, 500) = 100 * e^{-0.3*7071/7100} \approx 74.1$ dB, which is within the norm.

Point (9000, 9000), distance from source to point $d = 5657$:

$p_{eff}(9000, 9000) = 100 * e^{-0.3*5657/7100} \approx 78.7$ dB, which is within the norm.

The resulting variant of placement of sound signal sources and distribution of corresponding notification zones is shown in Fig. 2.

Thus, checking the sound pressure magnitude at the boundaries of the sound signal source's action zone and other important points of the warning territory ensures compliance with the criterion of the sound signals meeting the set normative values, that is, $p_{min} \le p_{eff}(x, y) < p_{max}$.

Conclusions. The results of the study allow us to conclude the effectiveness of using Laguerre-Voronoi diagrams for the optimal placement of sound signal sources in an emergency population warning system. The proposed algorithm for calculating coverage zones takes into account the action radii of warning sources, terrain features, and equipment acoustic characteristics to build the appropriate infrastructure for quality territorial coverage with the minimum number of sources and maximum coverage. The use of coverage zone methods, along with radical axes for forming convex polygon grids, ensures clear visualization and calculation accuracy. The use of acoustic characteristics provides the necessary data for analyzing compliance with the established sound pressure norms to ensure the quality of life of the population. The proposed approach can be used to improve emergency warning systems, enhance their reliability, and optimize installation costs as well as operational expenses for ensuring their functioning.

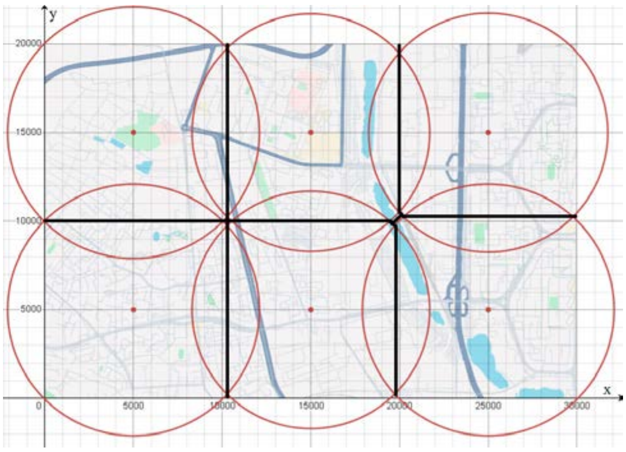

Fig. 2. Laguerre–Voronoi diagram of sound signal source placement and corresponding warning zones

Table 2

Results of optimizing the effective warning radii

Source	Coordinates	Radius before optimization	Radius after optimization	Absorption coefficient
1	(5000, 5000)	8060	7100	0.05
2	(15000, 5000)	7650	6700	0.1
3	(25000, 5000)	7220	7100	0.15
4	(5000, 15000)	8060	7100	0.05
5	(15000, 15000)	7650	6700	0.1
6	(25000, 15000)	7220	7100	0.15

The obtained results align well with prior studies on Voronoi-based optimizations [1–7], showcasing improved adaptability to heterogeneous urban settings. Compared to traditional grid-based placements, Laguerre-Voronoi diagrams demonstrate superior accuracy and efficiency in accounting for environmental variables such as terrain features and urban density.

Future research should explore:

1. Integration of dynamic environmental factors, like seasonal variations in acoustic absorption.
2. Scalability of the algorithm to large metropolitan areas with diverse building architectures.

The scientific novelty lies in applying Laguerre-Voronoi diagrams to optimize emergency alert systems by balancing effective coverage, regulatory compliance, and system costs. The proposed approaches have significant potential for further development and application in improving the infrastructure for placing sound signal sources in public safety systems.

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